Math 115

Winter 2017

Lecture 14

Factor Completely:

$2x^3 - 7x^2 - 15x = x(2x^2 - 7x - 15)$

$P = -30$

$S = -7$

$1, -30$

$2, -15$

$3, -10$

$5, -6$

-30

$2x^2 + 3x - 10x - 15$

$= x(2x+3) - 5(2x+3)$

$x(2x+3)(x-5)$

$60x^4 - 230x^3 + 200x^2$

$= 10x^2(6x^2 - 23x + 20)$

$P = 120$

$S = -23$

120

$-1, -120$

$-2, -60$

$-3, -40$

$-5, -24$

$-6, -20$

$-8, -15$

$-10, -12$

$6x^2 - 8x - 15x + 20$

$= 2x(3x-4) - 5(3x-4)$

$(3x-4)(2x-5)$

$10x^2(3x-4)(2x-5)$

$100x^3 - 16x$

$= 4x(25x^2 - 4)$

$= 4x(5x+2)(5x-2)$

$25x^2 - 4$

$(5x)^2 - (2)^2$

$A^2 - B^2$

$(A+B)(A-B)$

$= (5x+2)(5x-2)$

$54x^4y - 16xy^4$

$= 2xy(27x^3 - 8y^3)$

$= 2xy(3x-2y)(9x^2+6xy+4y^2)$

$27x^3 - 8y^3$

$(3x)^3 - (2y)^3$

$A^3 - B^3$

$= (A-B)(A^2+AB+B^2)$

$= (3x-2y)(9x^2+6xy+4y^2)$

$-6x^2 + 17x + 14$

$= -1(6x^2 - 17x - 14)$

$P = -84$

$S = -17$

-84

$6x^2 + 4x - 21x - 14$

$2x(3x+2) - 7(3x+2)$

$-1(3x+2)(2x-7)$

1, -84

2, -42

3, -28

4, -21

6, -14

7, -12

$-4x^2 + 20x - 25$

$= -(4x^2 - 20x + 25)$

$P = 100$

$S = -20$

100

$-1, -100$

$-2, -50$

$-4, -25$

$-5, -20$

$-10, -10$

$4x^2 - 10x - 10x + 25$

$= 2x(2x-5) - 5(2x-5)$

$= (2x-5)(2x-5)$

$= -(2x-5)^2$ ← Final Ans.

Perfect Square Trinomial

$A^2 + 2AB + B^2 = (A+B)^2$

$x^2 + 6x + 9 = (x+3)^2$

Verify that $2 \cdot x \cdot 3 = 6x$

$4x^2 + 20xy + 25y^2 = (2x+5y)^2$

Verify $2 \cdot 2x \cdot 5y = 20xy$

$49x^2 + 84x + 36$

$= (7x + 6)^2$

Verify $2 \cdot 7x \cdot 6 = 84x$

$64x^2 + 48xy + 9y^2 = (8x + 3y)^2$

$2 \cdot 8x \cdot 3y =$

$A^2 - 2AB + B^2 = (A - B)^2$

$9x^2 - 60x + 100 = (3x - 10)^2$

$2 \cdot 3x \cdot 10$

$144x^2 - 24x + 1 = (12x - 1)^2$

$2 \cdot 12x \cdot 1$

$625x^2 - 450xy + 9y^2$ Prime

$= (25x \quad 3y)^2$ Does not verify

$2 \cdot 25x \cdot 3y = 150xy$

$64x^2 - 112xy + 49y^2$

$= (8x - 7y)^2$

$2 \cdot 8x \cdot 7y = 112xy$

Zero-Factor Thrm:

If $A \cdot B = 0$, then $A = 0$ or $B = 0$ (Maybe both)

Solve $(x-7)(2x+5)=0$

By Z.F.T.

$x-7=0$ or $2x+5=0$

$x=7$ $\quad 2x=-5$ $\quad x=-\frac{5}{2}$

$\Rightarrow \left\{-\frac{5}{2}, 7\right\}$

Solve $(x+8)(x-4)(4x-3)=0$

RHS=0

LHS factored } Z.F.T. $x+8=0$ $\quad x-4=0$ $\quad 4x-3=0$

$x=-8$ $\quad x=4$ $\quad x=\frac{3}{4}$

$\left\{-8, \frac{3}{4}, 4\right\}$

Solve

① $(2x-7)(3x+5)=0$

By Zero-Product Rule

$2x-7=0$ or $3x+5=0$

$2x=7$ $\quad 3x=-5$

$x=\frac{7}{2}$ $\quad x=\frac{-5}{3}$

$\left\{-\frac{5}{3}, \frac{7}{2}\right\}$

② $4x(x-10)(4x+1)=0$

By Zero-Factor Rule

$4x=0$ or $x-10=0$ or $4x+1=0$

$x=\frac{0}{4}$ $\quad x=10$ $\quad 4x=-1$

$x=0$ $\quad x=\frac{-1}{4}$

$\left\{-\frac{1}{4}, 0, 10\right\}$

Solving Polynomial Eqns:

① Make sure RHS = 0

② Make sure LHS is completely factored

③ use Zero-Product Rule, and solve each factor

Solve $x^2-7x=-6$

$x^2-7x+6=0$

$(x-1)(x-6)=0$

→ By Z.F.T.,

$x-1=0$ or $x-6=0$

$x=1$ $\quad x=6$

$\{1, 6\}$

Solve

$x^2 - 4x = 5$

$x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

by Z.F.R.

$x+1=0$ or $x-5=0$

$x=-1$ $\quad x=5$ $\Rightarrow \{-1, 5\}$

Solve $x^2 + 9 = 6x$

$x^2 + 9 - 6x = 0$

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

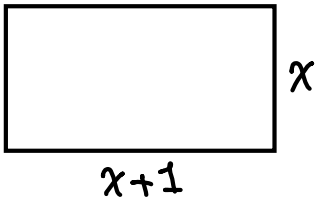
by Z.F.T.

$x-3=0$ $\quad \boxed{x=3}$ $\quad \{3\}$ Repeated.

Length and width of a rectangular carpert are two consecutive integers.

The area is $30\ m^2$.

Find its dimensions.



$A = 30$

$LW = 30$

$(x+1)x = 30$

$x^2 + x = 30$

$x^2 + x - 30 = 0$

$(x+6)(x-5) = 0$

1, 30
2, 15
3, 10
5, 6

by Z.F.T.

$x+6=0$ or $x-5=0$

~~$x=-6$~~ $\quad x=5$

5m by 6m.

Solve

$x^2 = 100$ $\quad x^2 - 100 = 0$

$x^2 - 10^2 = 0 \Rightarrow (x+10)(x-10) = 0$ by Z.F.T.

$A^2 - B^2$

$(A+B)(A-B)$ $\quad x+10=0$ or $x-10=0$

$x=-10$ $\quad x=10$

$\{\pm 10\}$

$4x^2 - 5 = 20$

$4x^2 - 5 - 20 = 0$

$4x^2 - 25 = 0$

$(2x)^2 - (5)^2 = 0 \rightarrow (2x+5)(2x-5)=0$ by Z.F.T.

$2x+5=0$ $\quad 2x-5=0$

$x=-5/2$ $\quad x=5/2$ $\quad \{\pm \frac{5}{2}\}$

$(x+3)(x+4) = 20$

① Foil & Simplify

$x^2 + 4x + 3x + 12 = 20$

$x^2 + 7x + 12 = 20$

② Make RHS=0, then factor the LHS

$x^2 + 7x + 12 - 20 = 0$

$x^2 + 7x - 8 = 0 \rightarrow (x+8)(x-1)=0$

Now use Z.F.T.

$x+8=0$ or $x-1=0$

$x=-8$ $\quad x=1$

$\{-8, 1\}$

Solve $(x-7)(x+1)=-16$

① Foil & Simplify $x^2+x-7x-7=-16$

$x^2-6x-7+16=0$

$x^2-6x+9=0$

② Factor LHS, use Z.F.T. $(x-3)(x-3)=0$ by Z.F.T.

$x-3=0$ $\boxed{x=3}$

$\{3\}$ Repeated Ans.

The Sum of Squares of two consecutive integers is 41. Find all such integers.

x & $x+1$

$x^2+(x+1)^2=41$

Solve

$x^2+(x+1)^2=41$

$x^2+(x+1)(x+1)=41$

$x^2+x^2+x+x+1=41$

$2x^2+2x+1-41=0$

$2x^2+2x-40=0$

Divide by 2

$x^2+x-20=0$

$(x+5)(x-4)=0$

By Z.F.T.

$x+5=0$ or $x-4=0$

$x=-5$, $x=4$

x	$x+1$
4	5
-5	-4

4 and 5 or -5 and -4

Solve:

$14x^2 - 3 = 21x - 3$

$14x^2 - \cancel{3} - 21x + \cancel{3} = 0$

$14x^2 - 21x = 0 \Rightarrow 2x^2 - 3x = 0$

Divide by 7

$x(2x - 3) = 0$

Use Z.F.T.

$x = 0$ or $2x - 3 = 0$

$\boxed{x = 0}$ $\boxed{x = \frac{3}{2}}$

$\left\{0, \frac{3}{2}\right\}$

Solve $3x^2 - 2x = 9 - 8x$

Make RHS = 0

$3x^2 - 2x - 9 + 8x = 0$

$3x^2 + 6x - 9 = 0 \Rightarrow x^2 + 2x - 3 = 0$

Divide by 3

$(x + 3)(x - 1) = 0$

by Z.F.R.

$x + 3 = 0, \; x - 1 = 0$

$\boxed{x = -3}$ $\boxed{x = 1}$

$\{-3, 1\}$

Solve

$12x^2 - 3x = 2 + 2x$

$12x^2 - 3x - 2 - 2x = 0$

$12x^2 - 5x - 2 = 0$

$P = -24$

$S = -5$

-24

1, 24

2, 12

3, -8

4, 6

$12x^2 + 3x - 8x - 2$

$3x(4x+1) - 2(4x+1)$

$(4x+1)(3x-2) = 0$

by Z.F.T.

$4x+1=0$ or $3x-2=0$

$4x=-1$ $3x=2$

$x=-\frac{1}{4}$ $x=\frac{2}{3}$

$\left\{-\frac{1}{4}, \frac{2}{3}\right\}$

Solve $15x^2 = 2 + 7x$

$15x^2 - 2 - 7x = 0$

$15x^2 - 7x - 2 = 0$

$P = -30$

$S = -7$

-30

1, -30

2, -15

3, -10

5, -6

$15x^2 + 3x - 10x - 2$

$3x(5x+1) - 2(5x+1)$

$(5x+1)(3x-2) = 0$

Z.F.T.

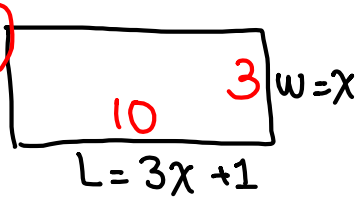
$5x+1=0$ or $3x-2=0$

$x=-\frac{1}{5}$ $x=\frac{2}{3}$

$\left\{-\frac{1}{5}, \frac{2}{3}\right\}$

Area of a rectangle is 30 ft^2.
The length is 1 ft longer than 3 times its width.
Find its dimensions.

3ft by 10ft



$A = 30$

$LW = 30$

$(3x+1)x = 30$

$3x^2 + x = 30$

$3x^2 + x - 30 = 0$

$3x^2 - 9x + 10x - 30$

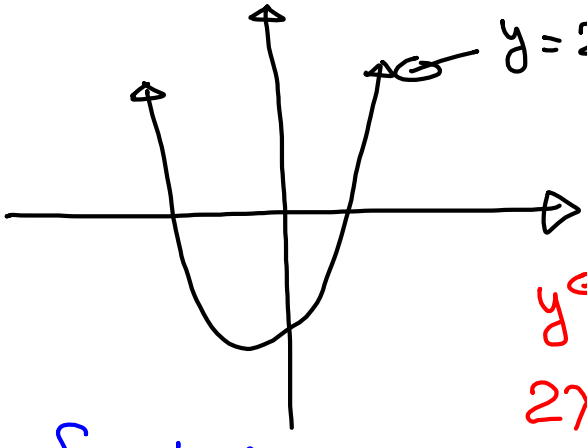
$3x(x-3) + 10(x-3) = (x-3)(3x+10)$

$P = -90$, $S = 1$: -90

$-1, 90$
$-2, 45$
$-3, 30$
$-5, 18$
$-6, 15$
$-9, 10$

$(x-3)(3x+10) = 0$ by Z.F.T.

$x = 3$, $x = -\frac{10}{3}$



$y = 2x^2 + x - 10$

Y-Int $\rightarrow (0, -10)$

X-Int $\rightarrow (\ , 0)$

$y = 0$

$2x^2 + x - 10 = 0$

$(2x+5)(x-2) = 0$

$+5x$
$-4x$

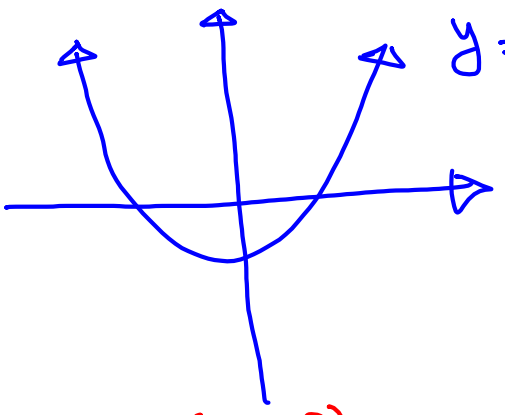
by Z.F.T.

$2x + 5 = 0$ $\quad x - 2 = 0$

$x = -5/2$ $\quad x = 2$

Final Ans

$(2, 0)$ & $(-5/2, 0)$



$y = 2x^2 + 3x - 9$

① Find Y-Int $(0, \ )$

② Find x-Int $(\ , 0)$

x-Int $(\frac{3}{2}, 0)$ & $(-3, 0)$

Y-Int $(0, -9)$

$y = 0$

$2x^2 + 3x - 9 = 0$

$(2x-3)(x+3) = 0$

$-3x$

$6x$

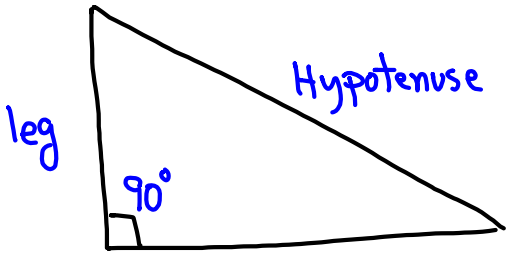
By Z.F.T.

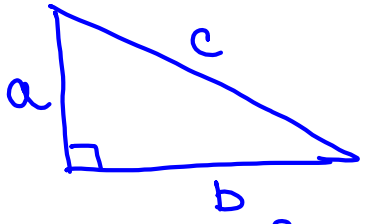
$2x - 3 = 0$ $\quad x + 3 = 0$

$x = \frac{3}{2}$ $\quad x = -3$

Exam 3

- Monday 6:00 – 7:40, come as early as 5:50.
- Review exam 1 & exam 2
- know exponential rules, Long Division
- Factoring, Solving Polynomial eqn.
- Factoring Project is due.
- This Sunday → 9:00 - 11:00 in the lobby of G5-Building.

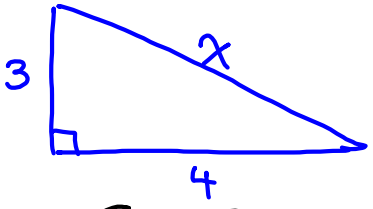




$a^2 + b^2 = c^2$

Pythagorean thm

Find x



$\{5\}$

by Pythagorean thm

$3^2 + 4^2 = x^2$

$9 + 16 = x^2$

$25 = x^2$

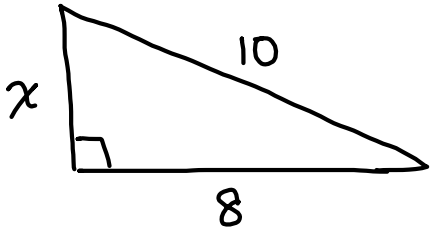
$x^2 = 25$

$x^2 - 25 = 0$

$(x+5)(x-5) = 0$

by Z.F.T.

~~$x = -5$~~, $x = 5$



Find x.

$\{6\}$

by Pythagorean thm

$x^2 + 8^2 = 10^2$

$x^2 + 64 = 100$

$x^2 + 64 - 100 = 0$

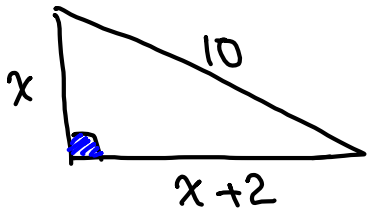
$x^2 - 36 = 0$

$(x+6)(x-6) = 0$

by Z.F.T.

~~$x = -6$~~, $x = 6$

Find x:



Right Triangle

Pythagorean thm

$x^2 + (x+2)^2 = 10^2$

$x^2 + (x+2)(x+2) = 100$

$x^2 + x^2 + 2x + 2x + 4 - 100 = 0$

$2x^2 + 4x - 96 = 0$

Divide by 2

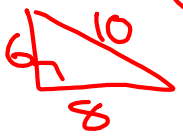
$x^2 + 2x - 48 = 0$

$(x+8)(x-6) = 0$

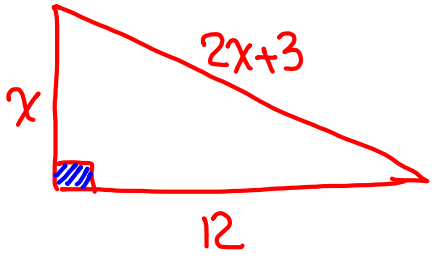
by Z.F.T.

~~-8~~ , 6

$\{6\}$



Find x



Right Triangle

Pythagorean thm

$x^2 + 12^2 = (2x+3)^2$

$x^2 + 144 = (2x+3)(2x+3)$

$x^2 + 144 = 4x^2 + 12x + 9$

$4x^2 + 12x + 9 = x^2 + 144$

$4x^2 + 12x + 9 - x^2 - 144 = 0$

$3x^2 + 12x - 135 = 0$

Divide by 3

$x^2 + 4x - 45 = 0$

$(x+9)(x-5) = 0$

by Z.F.T.

~~-9~~ & 5

$\{5\}$